Section 14.7: Maximum and Minimum Values

What We’ll Learn In Section 14.7…

1. How to find the local maximum and minimum of a 2-variable function
2. How to find the absolute maximum and minimum of a 2-variable function
3. Other Max and Min Problems

1. How to find the local maximum and minimum of a 2-variable function

Definition of Local Maximum: A point (a, b) is a local maximum of a 2-variable function f if…
there is a disk D in the domain of f centered at (a, b) such that
$f(x, y) \leq f(a, b)$ for all points $(x, y) \in D$.

This means that the point $(a, b, f(a, b))$ is the highest point on the graph of f among all points near it.

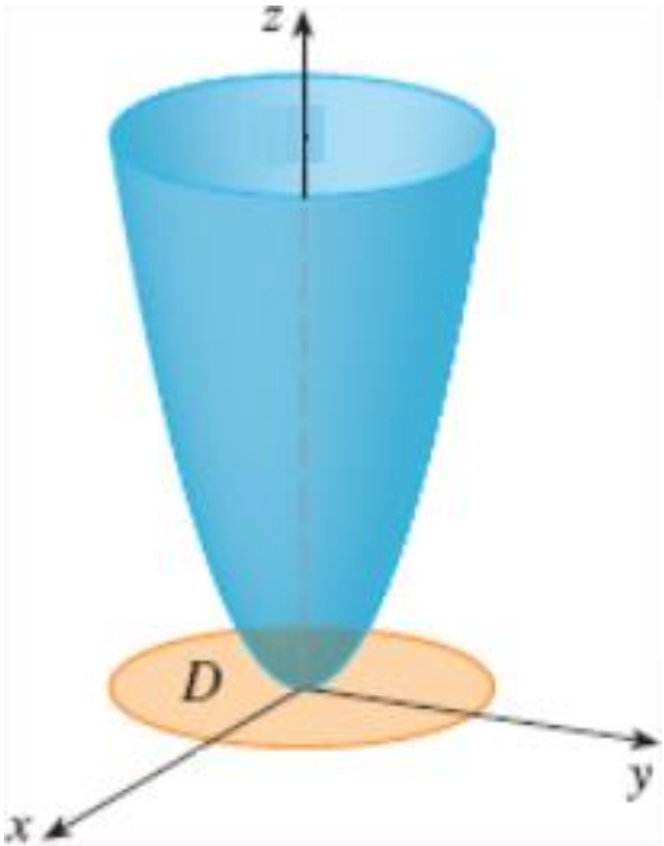
1. How to find the local maximum and minimum of a 2-variable function

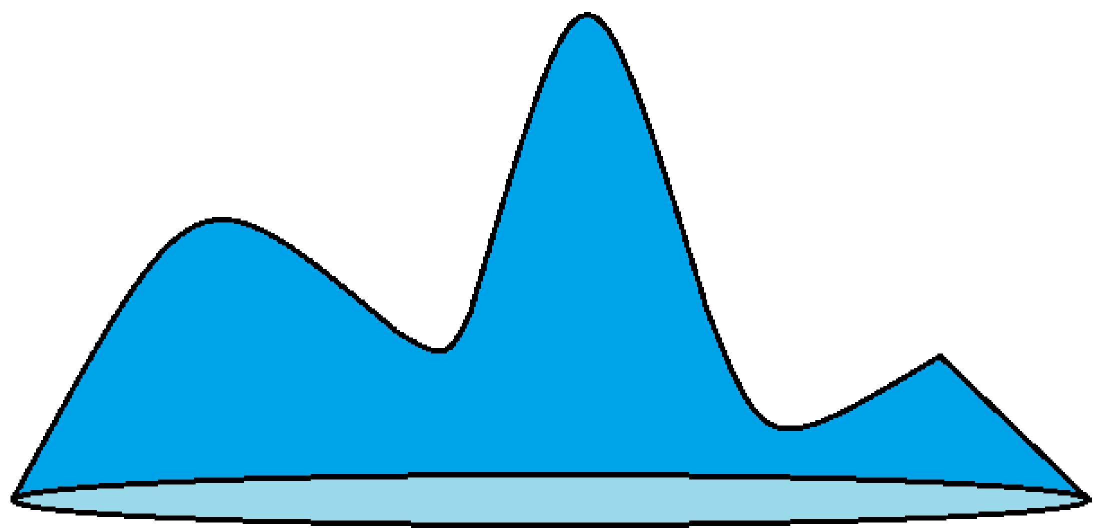
<u>Definition of Local Minimum</u>: A point (a,b) is a <u>local minimum</u> of a 2-variable function f if… there is a disk D in the domain of f centered at (a,b) such that
$f(x,y) \geq f(a,b)$ for all points $(x,y) \in D$.

This means that the point $(a,b,f(a,b))$ is the lowest point on the graph of f among all points near it.

1. How to find the local maximum and minimum of a 2-variable function

<u>Definition of Local Maximum/Local Minimum</u>:





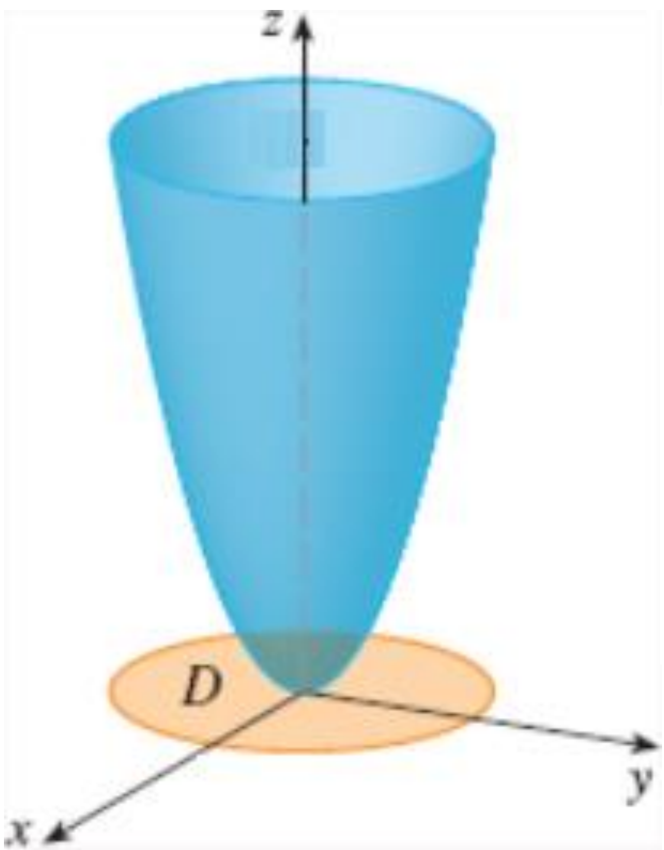
1. How to find the local maximum and minimum of a 2-variable function

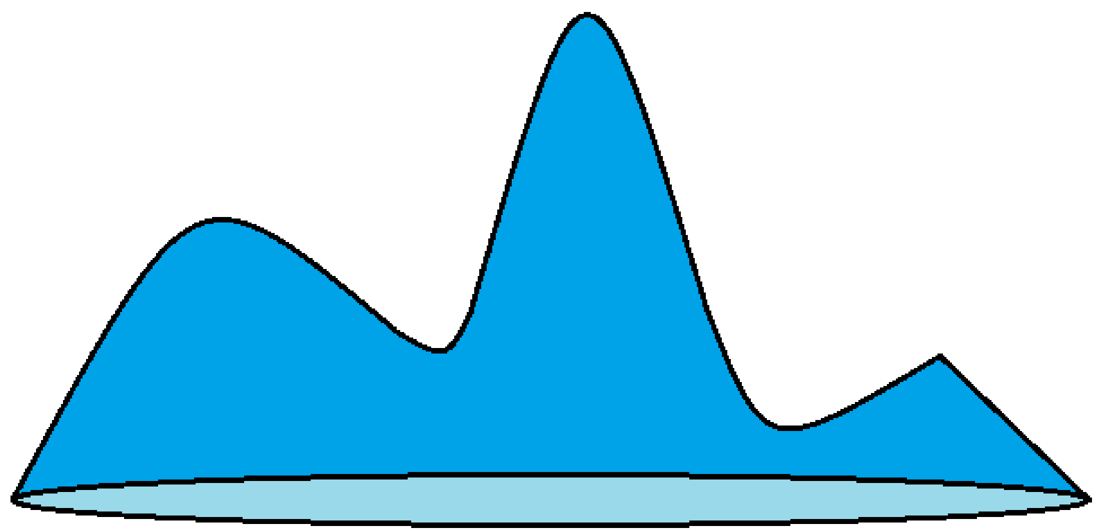
<u>Definition of Critical Point</u>: A point (a, b) is a <u>critical point</u> of a 2-variable function f if…
(a, b) is in the interior of the domain of f and either

1. $f_x(a, b) = 0$ AND $f_y(a, b) = 0$ or
2. if either $f_x(a, b) = DNE$ OR $f_y(a, b) = DNE$

1. How to find the local maximum and minimum of a 2-variable function

Definition of Critical Point:





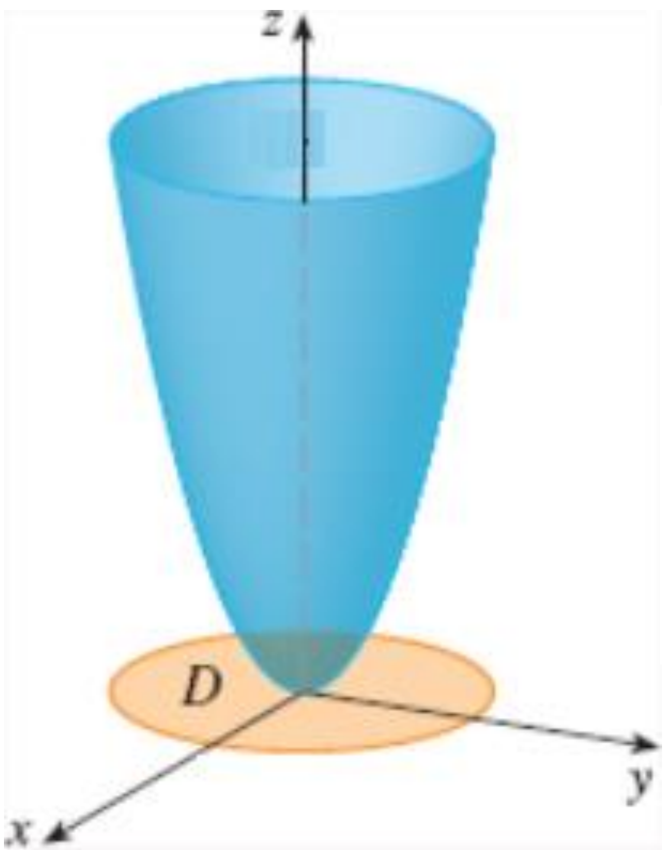
1. How to find the local maximum and minimum of a 2-variable function

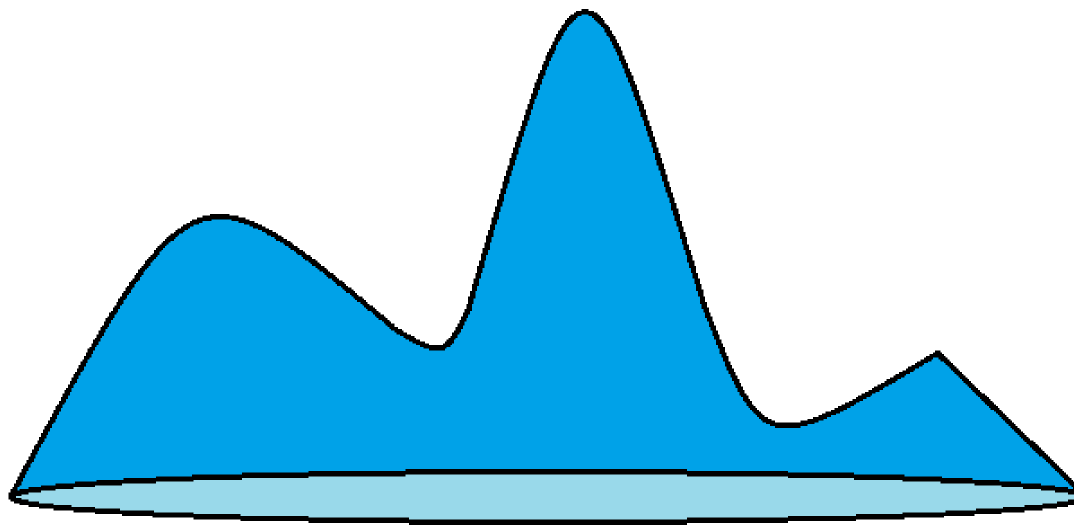
Theorem: If f has a local maximum or minimum at (a, b) and the first order partial derivatives exist there, then $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

- That is, local max and mins are critical points, but not the other way around.
- So in order to find local max and mins, first find the critical points. Then the local max and mins will be among the critical point you found.

1. How to find the local maximum and minimum of a 2-variable function

Theorem: Local max and mins are critical points.





1. How to find the local maximum and minimum of a 2-variable function

<u>Second Derivatives Test</u>

Suppose the second derivatives of a 2-variable function f are continuous on a disk with center (a, b), and suppose that (a, b) is a critical point of f.

Let $D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - \left[f_{xy}(a, b)\right]^2$

a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum
b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum
c) If $D < 0$, then $f(a, b)$ is a saddle point
d) If $D = 0$, this test gives no information

1. How to find the local maximum and minimum of a 2 variable function

<u>Ex 1 (book example 3):</u>

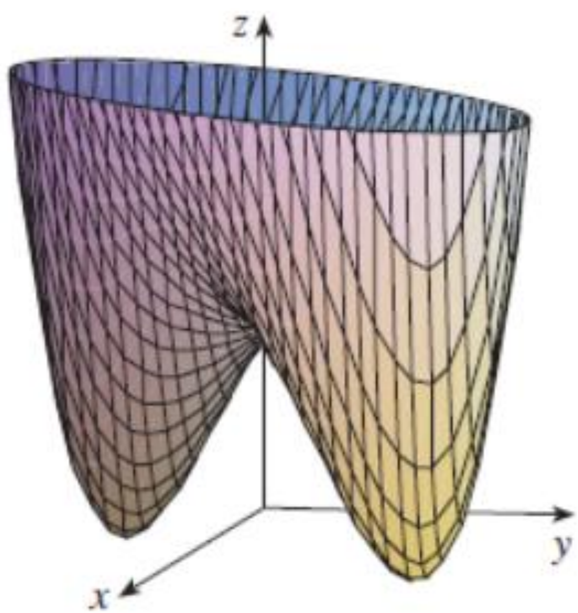
Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$

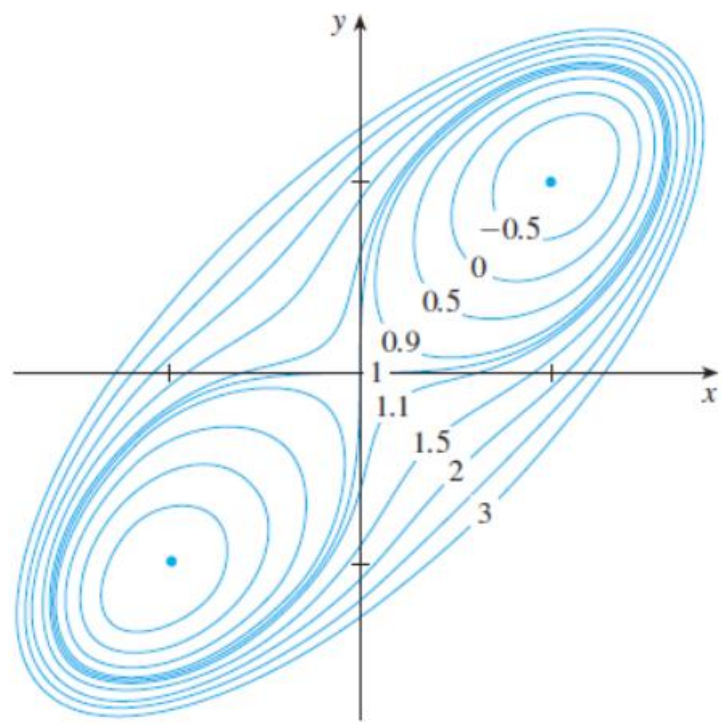
1. How to find the local maximum and minimum of a 2-variable function

<u>Ex 1 (book example 3):</u>

Find the local maximum and minimum values and saddle points of $f(x,y) = x^4 + y^4 - 4xy + 1$

$z = x^4 + y^4 - 4xy + 1$





2. How to find the absolute maximum and minimum of a 2-variable function

Definition of Absolute Maximum: A point (a, b) is an absolute maximum of a 2-variable function f if… $f(x, y) \leq f(a, b)$ for all points (x, y) in the domain of f.

This means that the point $(a, b, f(a, b))$ is the highest point on the graph of f.

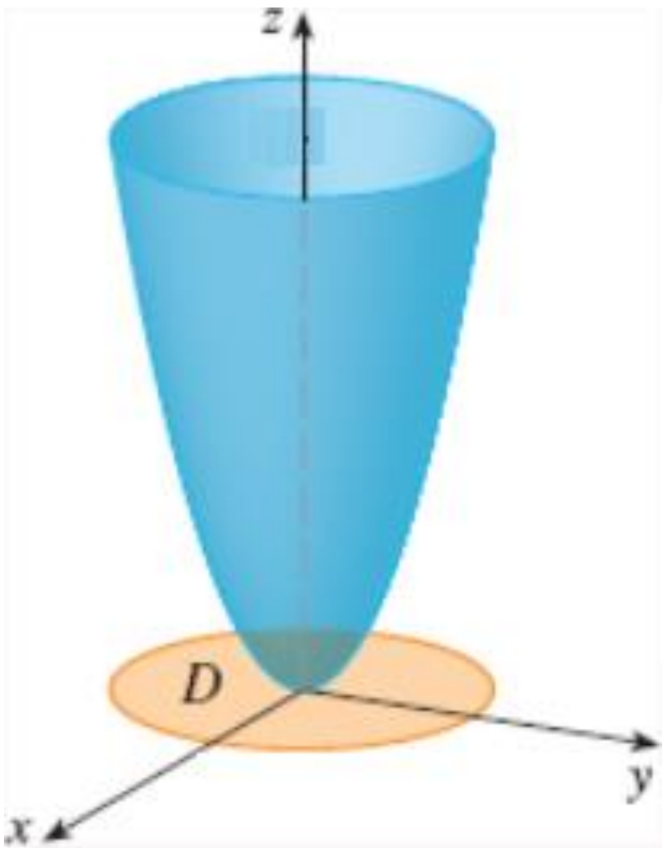
2. How to find the absolute maximum and minimum of a 2-variable function

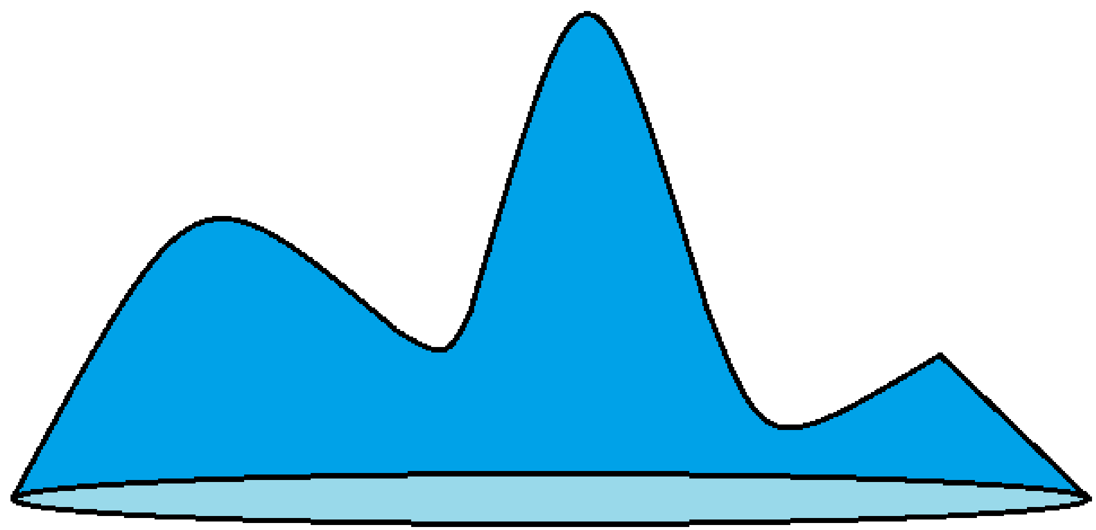
<u>Definition of Absolute Minimum</u>: A point (a,b) is an <u>absolute minimum</u> of a 2-variable function f if… $f(x,y) \geq f(a,b)$ for all points (x,y) in the domain of f.

This means that the point $(a,b,f(a,b))$ is the lowest point on the graph of f.

2. How to find the absolute maximum and minimum of a 2-variable function

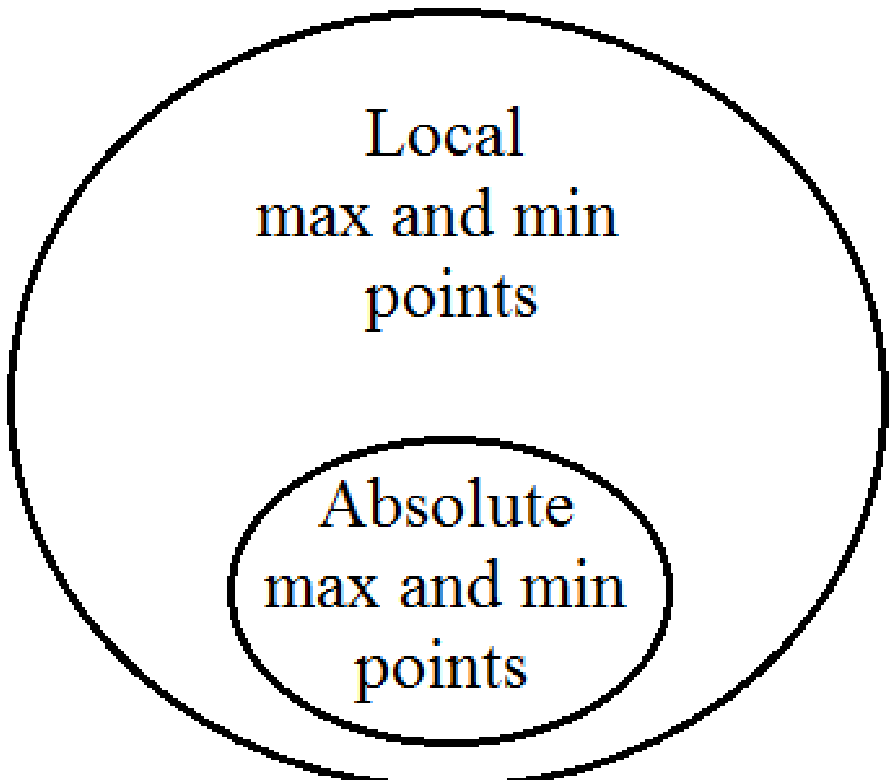
<u>Definition of Absolute Maximum/Absolute Minimum</u>:





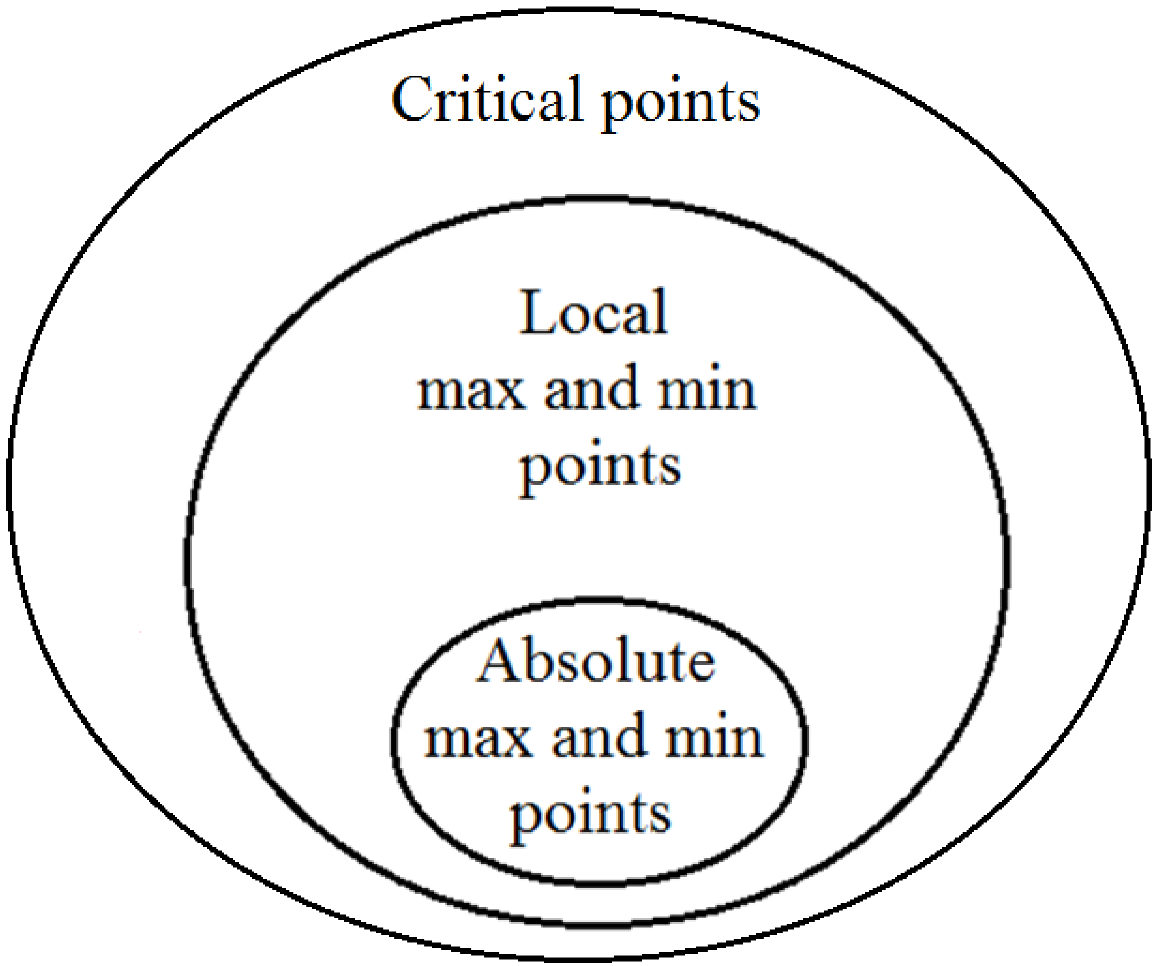
2. How to find the absolute maximum and minimum of a 2-variable function

Definition of Absolute Maximum/Absolute Minimum:



2. How to find the absolute maximum and minimum of a 2-variable function

Definition of Absolute Maximum/Absolute Minimum:



2. How to find the absolute maximum and minimum of a 2-variable function

Extreme Value Theorem for Functions of Two Variables:
If f is continuous on a closed, bounded set D in $\mathbb{R}^2$, then f attains an absolute maximum value $f(x_1, y_1)$ and an absolute minimum value $f(x_2, y_2)$ at some points (x_1, y_1) and (x_2, y_2) in D.

Notes:

- A closed set is a set that includes all of its boundary
- If you can draw a circle that completely encloses the set, then the set is bounded

2. How to find the absolute maximum and minimum of a 2-variable function

Extreme Value Theorem for Functions of Two Variables: If f is continuous on a closed, bounded set D in $\mathbb{R}^2$, then f attains an absolute maximum value $f(x_1, y_1)$ and an absolute minimum value $f(x_2, y_2)$ at some points (x_1, y_1) and (x_2, y_2) in D.

Notes:

- A closed set is a set that includes all of its boundary
- If you can draw a circle that completely encloses the set, then the set is bounded

2. How to find the absolute maximum and minimum of a 2-variable function

<u>To find the absolute max and min values of a continuous 2-variable function f on a closed, bounded set D</u>:

1. Find the values of f at the critical points of f in D
2. Find the extreme values of f on the boundary of D
3. The largest of the values from steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

2. How to find the absolute maximum and minimum of a 2-variable function

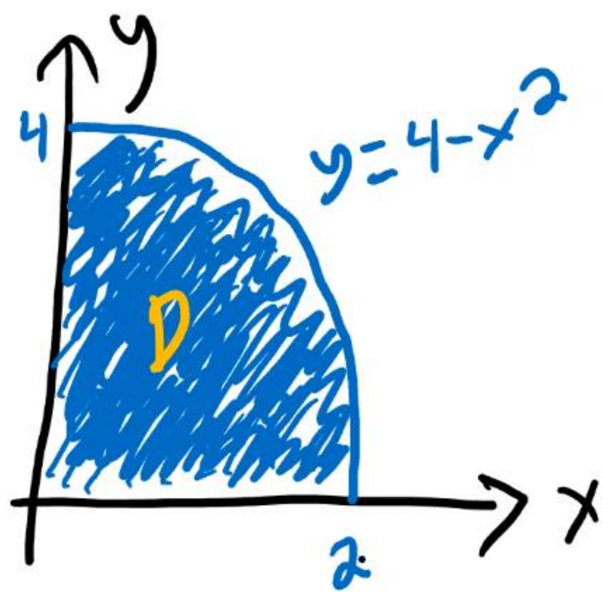
Ex 2 (book example 7):

Find the absolute maximum and minimum values of the function $f(x,y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x,y) \mid 0 \le x \le 3\,, 0 \le y \le 2\}$.

2. How to find the absolute maximum and minimum of a 2-variable function

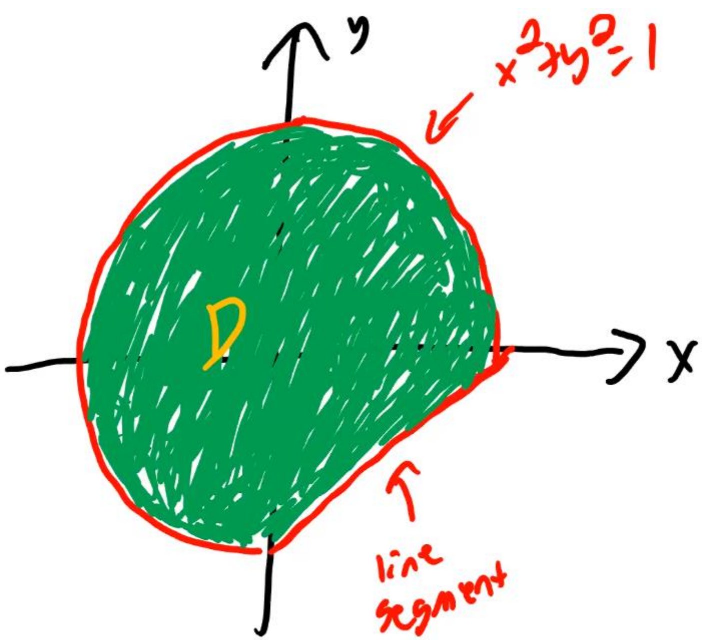
<u>Ex 3</u>:

Find the absolute maximum and minimum values (and where they occur) of the function $f(x,y) = x^2 - xy + y$ on the region D below.



2. How to find the absolute maximum and minimum of a 2-variable function

Ex 4 (book example 7):
Find the absolute maximum and minimum values (and where they occur) of the function
$f(x, y) = x^2 + xy + y^2$ on the region D below.



3. Other Max and Min Problems

Ex 5 (book example 5):

Find the shortest distance from the point $(1, 0, -2)$ to the plane $x + 2y + z = 4$.

3. Other Max and Min Problems

<u>Ex 6 (book example 6):</u>

A rectangular box without a lid is to be made from $12\ m^3$ of cardboard. Find the maximum volume of such a box.